Two-dimensional Array Beam Scanning via Externally and Mutually Injection Locked Coupled Oscillators

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Abstract - **The use of arrays of voltage-controlled oscillators coupled to nearest neighbors have been proposed as a means of controlling the aperture phase of one and two-dimensional phased array antennas. It has been demonstrated both theoretically and experimentally that one may achieve linear distributions of phase across a linear array aperture by injection locking to an external oscillator the end oscillators of an array of a mutually injection locked array of oscillators. These linear distributions cause steering of the radiated beam. It is demonstrated theoretically here that one may achieve beam steering in a similar manner in two dimensions by injecting appropriately phased signals into the perimeter oscillators of a two-dimensional array. The analysis is based on a continuum representation of the phase previously developed in the context of beam steering via tuning of the perimeter oscillators**

spatial power combining

I. INTRODUCTION

Some years ago, Stephan [1] proposed an approach to one dimensional (linear) phased array beam steering which requires only a single phase shifter. This involves the use of a linear array of voltage-controlled electronic oscillators coupled to nearest neighbors. The oscillators are mutually injection locked by controlling their coupling and tuning appropriately.[2][3] Stephan's approach consists of deriving two signals from a master oscillator, one signal phase shifted with respect to the other by means of a single phase shifter. These two signals are injected into the end oscillators of the array as shown in Figure 1. The result is a linear phase progression across the oscillator array. Thus, if radiating elements are connected to each oscillator and spaced uniformly along a line, they will radiate a beam at an angle to that line determined by the phase gradient which is, in turn, determined by the phase difference between the injection signals. The beam direction is therefore controlled by adjusting this phase difference.

Recently, Pogorzelski and York presented a formulation which facilitates theoretical analysis of the above beam steering technique.[4][5] This was subsequently applied by Pogorzelski in analysis of two-dimensional beam steering using perimeter detuning of a coupled oscillator array.[6] The formulation is based on a continuum model in which the oscillator phases are represented by a continuous function satisfying a partial differential equation of diffusion type. This equation can be solved via the Laplace transform and the resulting solution exhibits the dynamic behavior of the array as the beam is steered.

Stephan's beam steering technique can be similarly generalized to two-dimensional arrays in which the beam control signals are applied to the oscillators on the perimeter of the array. In this paper the continuum model for this two-dimensional case is developed and the dynamic solution for the corresponding aperture phase function is obtained. The corresponding behavior of the resulting far-zone radiation pattern is displayed as well.

II. THE TWO-DIMENSIONAL CONTINUUM MODEL

Consider a 2M+1 by 2N+1 rectangular array of coupled voltage-controlled oscillators. Let the oscillators be indexed by i and j. Suppose that externally derived signals are also injected into the oscillators indexed by pq. By applying Adler's theory of the dynamics of injection locking [7], it can be shown that the dynamic behavior of such an array is determined by a system of simultaneous differential equations which are first order in time. Specifically, these governing equations are

$$\frac{d\theta_{ij}}{dt} = \omega_{tune,ij} - \sum_{m=-M}^{M}\sum_{n=-N}^{N} \Delta\omega_{lock;ij,mn} \sin(\Phi_{ij,mn} + \theta_{ij} - \theta_{mn}) - \sum_{p=-M}^{M}\sum_{q=-N}^{N} \Delta\omega_{inj;ij,pq} \sin(\Phi_{ij,pq} + \theta_{ij} - \theta_{pq}) \ , \qquad (1)$$

where $\omega_{tune,ij}$ is the free running frequency of oscillator ij, $\Phi_{ij,mn}$ is the phase associated with the coupling between oscillators ij and mn in the array, and $\Delta\omega_{lock;ij,mn}$ is the locking range associated with that coupling and is given by

$$\Delta\omega_{lock,ij,mn} = \frac{\varepsilon_{ij,mn}\omega_{tune,ij}}{2Q}\frac{\alpha_{mn}}{\alpha_{ij}} \ , \qquad (2)$$

where α_{ij} is the amplitude of the output signal of the ij^{th} oscillator, $\varepsilon_{ij,mn}$ sets the strength of the coupling, and Q is the quality factor of the oscillators. $\Delta\omega_{inj;ij,pq}$ are the locking ranges associated with the external injection signals, $\Phi_{ij,pq}$ is the associated coupling phase, and θ_{pq} is the phase of each external oscillator signal. The phase, θ_{ij}, is the phase of the ij^{th} oscillator; that is,

$$\theta_{ij} = \omega_{ref}t + \phi_{ij} \ , \qquad (3)$$

where ω_{ref} is the reference frequency for defining the phase, ϕ_{ij}, of each oscillator. If the oscillators are coupled only to nearest neighbors, equation (1) simplifies to

$$\frac{d\phi_{ij}}{dt} = \omega_{tune,ij} - \omega_{ref} - \sum_{\substack{m=i-1 \\ m\neq i}}^{i+1}\sum_{\substack{n=j-1 \\ n\neq j}}^{j+1} \Delta\omega_{lock,ij,mn} \sin(\Phi_{ij,mn} + \phi_{ij} - \phi_{mn}) - \sum_{p=-M}^{M}\sum_{q=-N}^{N} \Delta\omega_{inj,ij,pq} \sin(\Phi_{ij,pq} + \phi_{ij} - \phi_{pq}) \ . \qquad (4)$$

Taking the coupling phase to be zero and assuming that the phase differences between adjacent oscillators are small, the sine functions may be replaced by their arguments. Then, following the reasoning detailed in earlier papers [4][5], it is noted that, if all if the locking ranges are equal, equation (4) becomes

$$\frac{d\phi_{ij}}{dt} = \omega_{tune,ij} - \omega_{ref} - \Delta\omega_{lock}\left[(\phi_{ij} - \phi_{i-1,j-1}) + (\phi_{ij} - \phi_{i-1,j+1}) + (\phi_{ij} - \phi_{i+1,j-1}) + (\phi_{ij} - \phi_{i+1,j+1})\right]$$
$$- \sum_{q=-M}^{M} \sum_{p=-N}^{N} \Delta\omega_{inj}(\phi_{ij} - \phi_{pq}) \qquad (5)$$
$$= \omega_{tune,ij} - \omega_{ref} + \Delta\omega_{lock}\left[\phi_{i-1,j-1} + \phi_{i-1,j+1} + \phi_{i+1,j-1} + \phi_{i+1,j+1} - 4\phi_{ij}\right] - \sum_{q=-M}^{M} \sum_{p=-N}^{N} \Delta\omega_{inj}(\phi_{ij} - \phi_{pq}).$$

As before, the quantity in brackets on the right side of (5) is seen to be a discrete approximation to the Laplacian operator in two dimensions. Indexing the oscillators by integer values of continuous variables x and y, and representing the oscillator phases by the continuous function $\phi(x,y;\tau)$, one arrives at the partial differential equation,

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - V(x,y)\phi - \frac{\partial\phi}{\partial\tau} = -\frac{\omega_{tune} - \omega_{ref}}{\Delta\omega_{lock}} - V(x,y)\phi_{inj}(x,y;\tau) , \qquad (6)$$

for the phase function, $\phi(x,y;\tau)$, where τ is a dimensionless time measured in inverse locking ranges; that is,

$$\tau = \Delta\omega_{lock}\, t \qquad (7)$$

and $V(x,y)$ describes the strength and distribution of injection signals over the array.

A unit cell one unit square is associated with each oscillator so that the array extent is given by

$$\begin{aligned} -\left(M + \frac{1}{2}\right) \le i \le \left(M + \frac{1}{2}\right), \\ -\left(N + \frac{1}{2}\right) \le j \le \left(N + \frac{1}{2}\right). \end{aligned} \qquad (8)$$

As was shown in earlier work[4][5][6], the study of the dynamic behavior of the array is now a matter of solving equation (6) subject to Neumann boundary conditions at $x = \pm\left(a + \frac{1}{2}\right)$ and at $y = \pm\left(b + \frac{1}{2}\right)$ where a and b correspond to index values i=M and j=N denoting the perimeter oscillators in the array.

III. BEAMSTEERING IN TWO DIMENSIONS

Consider a rectangular array with (2M+1)(2N+1) oscillators extending over the range

$$-\left(a+\frac{1}{2}\right) \le x \le \left(a+\frac{1}{2}\right),$$
$$-\left(b+\frac{1}{2}\right) \le y \le \left(b+\frac{1}{2}\right). \tag{9}$$

Beamsteering requires that the aperture constant phase surface be planar but with normal tilted with respect to the normal to the aperture. The beam is directed normal to this tilted constant phase surface and is thus steered away from the aperture normal direction. To obtain such an aperture phase distribution, we begin by considering the following situation as a preliminary. First, all oscillators are tuned to the external injection frequency which is take to be the reference frequency in the differential equation. Extending Stephan's approach to two dimensions implies injection signals at the oscillators on the perimeter of the array. Thus *V(x,y)* takes the form

$$V(x,y) = C\,P(x) + C\,Q(y), \tag{10}$$

where

$$C = \frac{\Delta\omega_{inj}}{\Delta\omega_{lock}}, \tag{11}$$

and

$$C\,P(x) = \Omega_{x_1}\delta(x - x_1') + \Omega_{x_2}\delta(x - x_2'),$$
$$C\,Q(y) = \Omega_{y_1}\delta(y - y_1') + \Omega_{y_2}\delta(y - y_2'). \tag{12}$$

The use of the Dirac delta function in this context was considered in detail previously and will therefore be used here without further discussion.[4][5] For perimeter injection x_1', x_2', y_1', and y_2' are set to -a, a, -b, and b, respectively but will be left general for now. It is important to recognize that *P* and *Q* set the strength and distribution of the injection signals but their phase enters the formulation through the function ϕ_{inj} in (6). Substituting (10) into (6) results in

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - CP(x)\phi - CQ(y)\phi - \frac{\partial\phi}{\partial\tau} = -CP(x)\phi_{inj}(x,y;\tau) - CQ(y)\phi_{inj}(x,y;\tau). \tag{13}$$

Now temporarily consider the homogeneous equation,

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - CP(x)\phi - CQ(y)\phi - \frac{\partial\phi}{\partial\tau} = 0 \quad . \tag{14}$$

The plan of attack is to solve this equation with Neumann boundary conditions for the eigenfunctions and to write the Green's function as a series expansion in these eigenfunctions. Finally, the Green's function will be used to solve (13). Laplace transformation of (14) with respect to τ results in

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}-CP(x)f-CQ(y)f-sf=0 \ , \tag{15}$$

where f is the Laplace transform of ϕ. Now, let $f(x,y)=X(x,s_x)Y(y,s_y)$. Separation of variables then yields

$$\begin{aligned} X''-CPX-s_xX&=0\,,\\ Y''-CQY-s_yY&=0\,, \end{aligned} \tag{16}$$

where $s=s_x+s_y$ and the double primes of course indicating second order differentiation with respect to the spatial variable. Substituting (12) into (16) now gives

$$\begin{aligned} X''-\Omega_{x_1}\delta(x-x_1')-\Omega_{x_2}\delta(x-x_2')-s_xX&=0\,,\\ Y''-\Omega_{y_1}\delta(y-y_1')-\Omega_{y_2}\delta(y-y_2')-s_yY&=0\,. \end{aligned} \tag{17}$$

These equations are now solved for the eigenfunctions satisfying Neumann boundary conditions. Because of the similarity of the two forms, the equation for X will be treated and the solution for Y will then follow by inspection. The solution is postulated in the form

$$\begin{aligned} X&=A_1\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x_1'\right)\right]\\ &\quad+A_2\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x_2'\right)\right], \quad -a-\frac{1}{2}\le x\le x_1'\,,\\ X&=A_1\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x_1'\right)\right]\\ &\quad+A_2\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x_2'\right)\right], \quad x_2'\le x\le x_1'\,,\\ X&=A_1\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x_1'\right)\right]\\ &\quad+A_2\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x_2'\right)\right], \quad x_2'\le x\le a+\frac{1}{2}\,. \end{aligned} \tag{18}$$

The constants, A_1 and A_2, as well as the eigenvalues, s_x, are determined by the slope discontinuity conditions at x_1' and x_2' which are obtained by integrating (17) across each of these discontinuity points. The resulting conditions are

$$
\begin{aligned}
X'\big|_{x_1'^-}^{x_1'^+} &= \Omega_{x_1} X(x_1'), \\
X'\big|_{x_2'^-}^{x_2'^+} &= \Omega_{x_2} X(x_2').
\end{aligned}
\tag{19}
$$

Application of these conditions to (18) yields

$$
\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0,
\tag{20}
$$

where

$$
\begin{aligned}
M_{11} &= \sqrt{s_x}\sinh\left[\sqrt{s_x}(2a+1)\right] + \Omega_{x_1}\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x_1'\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x_1'\right)\right], \\
M_{12} &= \Omega_{x_1}\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x_1'\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x_2'\right)\right], \\
M_{21} &= \Omega_{x_2}\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x_1'\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x_2'\right)\right], \\
M_{22} &= \sqrt{s_x}\sinh\left[\sqrt{s_x}(2a+1)\right] + \Omega_{x_2}\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}+x_2'\right)\right]\cosh\left[\sqrt{s_x}\left(a+\frac{1}{2}-x_2'\right)\right].
\end{aligned}
\tag{21}
$$

The determinant of the coefficients reduces to

$$
\begin{aligned}
\Delta = \sinh\left[\sqrt{s_x}(2a+1)\right]\Big\langle s_x \sinh\left[\sqrt{s_x}(2a+1)\right] &+ \frac{1}{2}\left(\Omega_{x_1}+\Omega_{x_2}\right)\sqrt{s_x}\cosh\left[\sqrt{s_x}(2a+1)\right] \\
&+ \frac{1}{2}\Omega_{x_1}\sqrt{s_x}\cosh\left[\sqrt{s_x}(2x_1')\right] + \frac{1}{2}\Omega_{x_2}\sqrt{s_x}\cosh\left[\sqrt{s_x}(2x_2')\right] \\
-\frac{1}{2}\Omega_{x_1}\Omega_{x_2}\sinh\left[\sqrt{s_x}(x_1'-x_2')\right]&\left\{\cosh\left[\sqrt{s_x}(2a+1+x_1'-x_2')\right] + \cosh\left[\sqrt{s_x}(x_1'+x_2')\right]\right\}\Big\rangle.
\end{aligned}
\tag{22}
$$

Setting this determinant equal to zero yields a transcendental equation for the eigenvalues, s_x, which can be solved numerically. Moreover, using (20) one can easily show that the corresponding eigenfunctions are given by (18) with

$$
\begin{aligned}
A_1 &= M_{12} = A_1^{(1)}, \\
A_2 &= -M_{22} = A_2^{(1)},
\end{aligned}
\tag{23}
$$

or

$$
\begin{aligned}
A_1 &= -M_{11} = A_1^{(2)} \, , \\
A_2 &= M_{21} = A_2^{(2)} \, .
\end{aligned} \tag{24}
$$

These two choices of constants yield the same eigenfunction to within a constant which of course becomes irrelevant upon normalization of the functions. Substitution of the constants into (18) gives the final result for $X(x,s_x)$ in the form,

$$
\begin{aligned}
X(x,s_x) = {} & \cosh\left[\sqrt{s_x}\left(2h_x - |x_2' - x|\right)\right] + \eta \cosh\left[\sqrt{s_x}\left(2h_x - |x_1' - x|\right)\right] \\
& \cosh\left[\sqrt{s_x}\left(x + x_2'\right)\right] + \eta \cosh\left[\sqrt{s_x}\left(x + x_1'\right)\right] \\
\Omega_{x_1} & \cosh\left[\sqrt{s_x}\left(h_x + x_1'\right)\right]\left\{\frac{1}{\sqrt{s_x}}\sinh\left[\sqrt{s_x}\left(h_x - x_1' - |x_2' - x|\right)\right] - \frac{1}{\sqrt{s_x}}\sinh\left[\sqrt{s_x}\left(h_x - x_2' - |x_1' - x|\right)\right]\right\} \\
\Omega_{x_2} & \cosh\left[\sqrt{s_x}\left(h_x - x_2'\right)\right]\left\{\frac{1}{\sqrt{s_x}}\sinh\left[\sqrt{s_x}\left(h_x + x_2' - |x_1' - x|\right)\right] - \frac{1}{\sqrt{s_x}}\sinh\left[\sqrt{s_x}\left(h_x + x_1' - |x_2' - x|\right)\right]\right\}\eta \, ,
\end{aligned} \tag{25}
$$

where $h_x = a + \dfrac{1}{2}$ and the choice of η equal to +1 or -1 corresponds to selecting (23) or (24). The choice becomes useful if the determinant (22) is zero by virtue of both elements of a row becoming zero. This renders both the eigenfunction and its normalization integral zero so that the expression for the normalized eigenfunction becomes indeterminate. When this happens, the other choice yields a determinate result. The normalization integrand which is the square of (25), while complicated, can be integrated in closed form yielding a closed form expression for each of the normalized eigenfunctions. A similar process provides the corresponding set of y dependent eigenfunctions. The products of the x and y dependent eigenfunctions are the eigenfunctions of the two-dimensional problem.

The above set of two-dimensional eigenfunctions form an orthonormal set suitable for representing the Green's function as a series. The familiar result is

$$
G(x,y;x',y';s) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{X(x',s_m)Y(y',s_n)X(x,s_m)Y(y,s_n)}{s - s_m - s_n} \quad , \tag{26}
$$

which is the solution of

$$
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - CP(x)\phi - CQ(y)\phi - \frac{\partial \phi}{\partial \tau} = \delta(x - x')\delta(y - y') \quad , \tag{27}
$$

with P and Q given by (12).

Returning now to (13), the Laplace transform of the inhomogeneous equation is

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}-CP(x)f-CQ(y)f-sf=-CP(x)f_{inj}(x,y;s)-CQ(y)f_{inj}(x,y;s)\,. \qquad (28)$$

The solution in terms of the Green's function is, of course, expressible as

$$f(x,y;s)=-C\int_{\left(-b-\frac{1}{2}\right)}^{\left(b+\frac{1}{2}\right)}\int_{\left(-a-\frac{1}{2}\right)}^{\left(a+\frac{1}{2}\right)}G(x,y;x',y';s)\left[P(x')f_{inj}(x',y';s)+Q(x')f_{inj}(x',y';s)\right]dx'dy'\,. \qquad (29)$$

Specifically, the necessary P, Q, and f_{inj} for beamsteering can be obtained as follows. From (28) one can infer that, if f_{inj} is linear in x and y, the steady state value of ϕ at the oscillators which are externally injected will be equal to ϕ_{inj}. Thus, to establish a planar phase across the array it is appropriate to use a function f_{inj} which is linear in x and y representing the desired planar phase variation. For a radiating aperture with element spacing, d, the function required to point the beam in a direction given by polar angles, (ϕ_0, θ_0), with respect to the array normal is, therefore,

$$f_{inj}(x,y;s)=\frac{1}{s}\left[-x\sin\theta_0\cos\phi_0-y\sin\theta_0\sin\phi_0\right]\frac{2\pi d}{\lambda}\,. \qquad (30)$$

Thus, $P(x)\,f_{inj} + Q(y)\,f_{inj}$ will be

$$\begin{aligned}P(x)f_{inj}+Q(y)f_{inj}=\frac{1}{s}\Big[&-\delta(x-x_1')x_1'\sin\theta_0\cos\phi_0-\delta(x-x_1')y\sin\theta_0\sin\phi_0\\&-\delta(x-x_2')x_2'\sin\theta_0\cos\phi_0-\delta(x-x_2')y\sin\theta_0\sin\phi_0\\&-\delta(y-y_1')x\sin\theta_0\cos\phi_0-\delta(y-y_1')y_1'\sin\theta_0\sin\phi_0\\&-\delta(y-y_2')x\sin\theta_0\cos\phi_0-\delta(y-y_2')y_2'\sin\theta_0\sin\phi_0\Big]\frac{2\pi d}{\lambda}\,,\end{aligned} \qquad (31)$$

where λ is the free space wavelength. The integrations in (29) involving the Dirac delta functions in P and Q can be carried out in closed form and the inverse Laplace transform is then merely a matter of evaluating the residues at the simple pole contained in each term of the series. If the injection signal phase is switched at time zero; i.e., contains a unit step function; the function f_{inj} will contain an s in its denominator. That is, it will have a simple pole at the origin of the s plane. This will give rise to a series of time independent terms in the inverse transform which represent the steady state solution for phase. Thus, the overall solution will have the form

$$\phi(x,y;\tau)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}R_{mn}(x,y)\left[1-e^{(s_m+s_n)\tau}\right]u(\tau)\,, \qquad (32)$$

where R_{mn} is the residue at $s=s_m+s_n$ and $u(\tau)$ is the unit step function. This series is very slowly convergent particularly at late times. However, the convergence rate can be vastly improved by using the steady state solution in the closed form,

$$\begin{aligned}\phi(x,y;\infty) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} R_{mn}(x,y)u(\tau) = \phi_{inj}(x,y) \\ &= \lim_{s\to 0}\left\{sf_{inj}(x,y,s)\right\} = -\left(x\sin\theta_0\cos\phi_0 + y\sin\theta_0\sin\phi_0\right)\frac{2\pi d}{\lambda}\end{aligned} \tag{33}$$

and rewriting (32) as,

$$\phi(x,y;\tau) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} R_{mn}(x,y)\left[-e^{(s_m+s_n)\tau}\right]u(\tau) - \left(x\sin\theta_0\cos\phi_0 + y\sin\theta_0\sin\phi_0\right)\frac{2\pi d}{\lambda}. \tag{34}$$

This form converges rapidly for all but the earliest times when the phase is essentially equal to its initial value which is, in this case, zero.

IV. A NUMERICAL EXAMPLE

Consider a 21 by 21 element square array with radiating elements spaced one half wavelength apart. Injection signals are assumed to be applied to the perimeter oscillators with phases appropriate to steering the beam to far-zone coordinates $\theta_0=30°$ and $\phi_0=-110°$ switched at time zero. Figure 2 illustrates the ensuing dynamic behavior of the oscillators phases. Note that during this transient period the phase surface is nonplanar which results in some aberration induced gain reduction and sidelobe distortion in the far-zone beam. Figure 3 traces the evolution of the beam shape during the beamsteering transient by displaying the locations of the beam peak and the three dB contour at a sequence of times from 0 to 45 inverse locking ranges in increments of 5 as noted on the graph. The effect of aberration on the beam shape is evident in the early stages of the transition. Figure 4 shows the effect of this aberration on the far-zone gain as a function of time. The curve labeled "Ideal Gain" includes the projected aperture loss but no aberration loss for comparison. In all cases discussed so far, the injection level is such that the external injection locking range is 70% of the inter-oscillator injection locking range; i.e., C as defined in equation (35) is 0.7. Figures 5 and 6 show the gain as a function of time for weaker (C=0.2) and stronger (C=10.0) injection levels, respectively. Note that the transient decays faster for higher injection levels but that the initial gain dip is deeper and the beam peak initially moves more erratically. Finally, Figure 7 shows the result of successive application of a sequence of beamsteering phases using the representation of Figure 3 and returning to the original C=0.7 level. Here again, as in Figure 3, the notation on the graph indicates a sequence of times, in this case from 0 to 397.5 inverse locking ranges in increments of 2.5.

V. CONCLUDING REMARKS

A beam scanning concept proposed by Stephan in the context of a one-dimensional phased array antenna has been generalized to two-dimensional arrays. A continuum formalism developed over the past several years and described in earlier publications was applied in analysis of the corresponding two-dimensional case and the resulting beamsteering dynamics illustrated via numerical computation. As described previously in terms of the one dimensional case [5], although when the steering phase shift is applied suddenly, the overall phase shift across the entire array is limited to less than ninety degrees, gradual change in the applied phase shift permits phase shifts limited only by the ninety degree inter-oscillator phase difference limit. Thus, the actual limit on phase shift across the entire array is ninety degrees times the number of oscillators less one. For half wavelength spacing of the radiating elements, this would seem to limit the beam scan to thirty degrees from normal, this limit may be removed by placing an additional oscillator between each radiating oscillator or by radiating the second harmonic frequency.[6] Finally, it is remarked that Stephan's scheme represents an alternative to the detuning method of Liao and York [8] described in reference [6], but appears to be somewhat more complicated to implement because of the need for linear phase progressions along the array edges as opposed to the constant tuning voltages used in the latter technique.

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Figure Captions

Figure 1. Stephan's single phase shifter beamsteering scheme.

Figure 2. Phase dynamics during beam steering.

Figure 3. Antenna beam peak and three dB contours.

Figure 4. Antenna gain during beam steering.

Figure 5. Antenna gain during beam steering for weak injection.

Figure 6. Antenna gain during beam steering for strong injection.

Figure 7. Sequential beam steering.

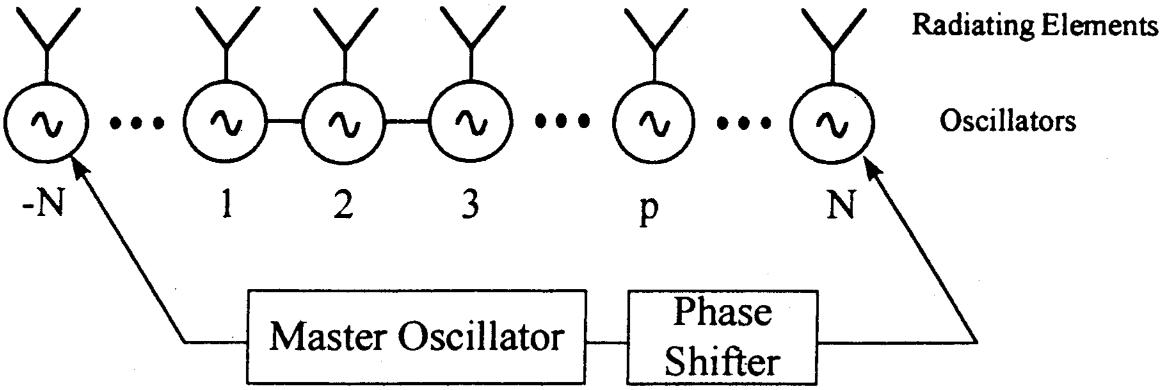

Figure 1. Stephan's single phase shifter beamsteering scheme.

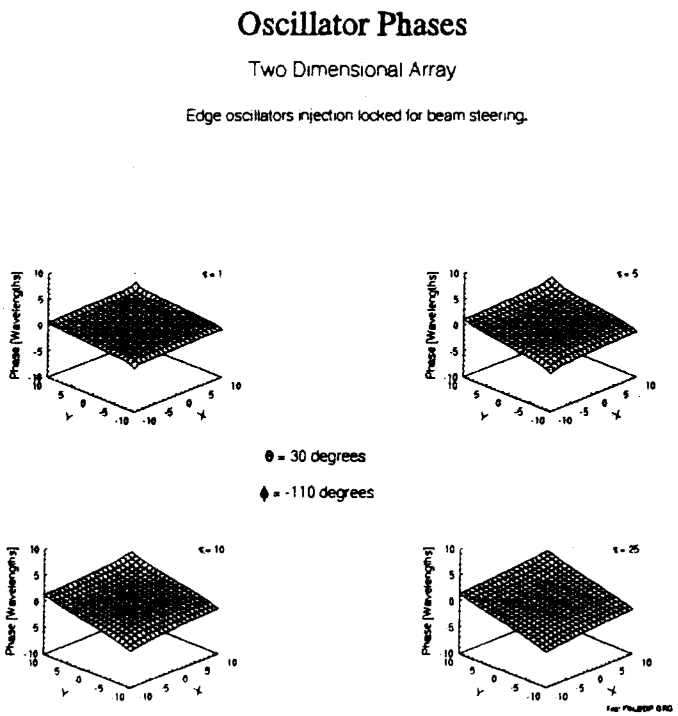

Figure 2. Phase dynamics during beam steering.

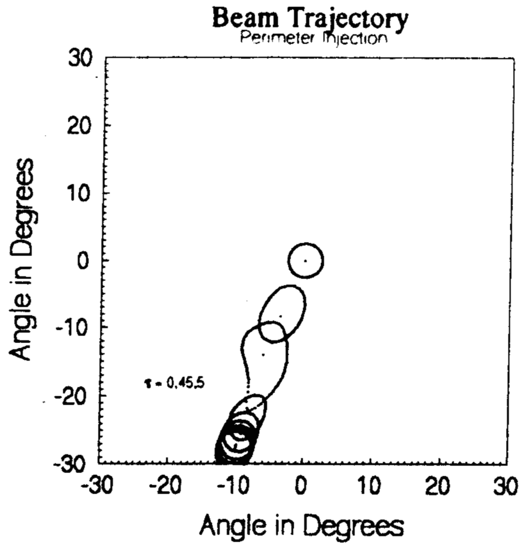

Figure 3. Antenna beam peak and three dB contours.

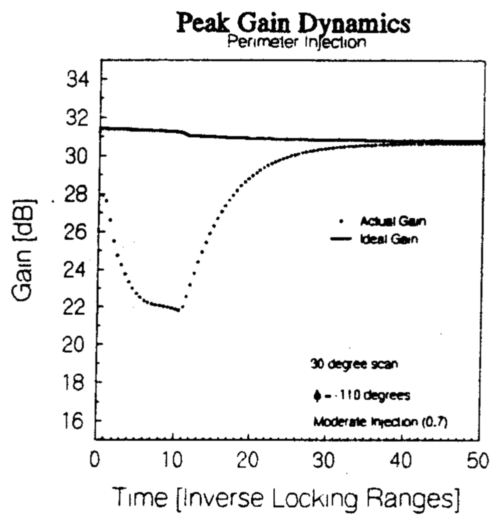

Figure 4. Antenna gain during beam steering.

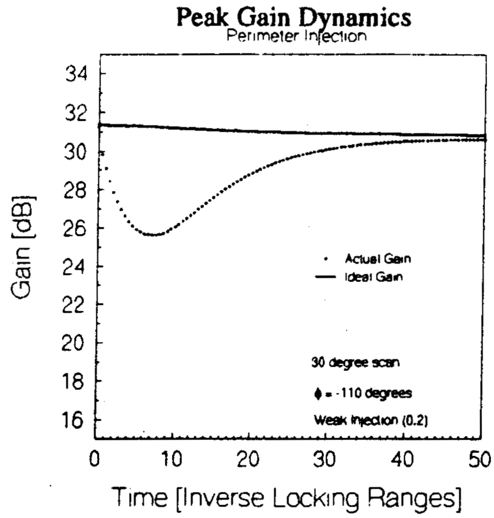

Figure 5. Antenna gain during beam steering for weak injection.

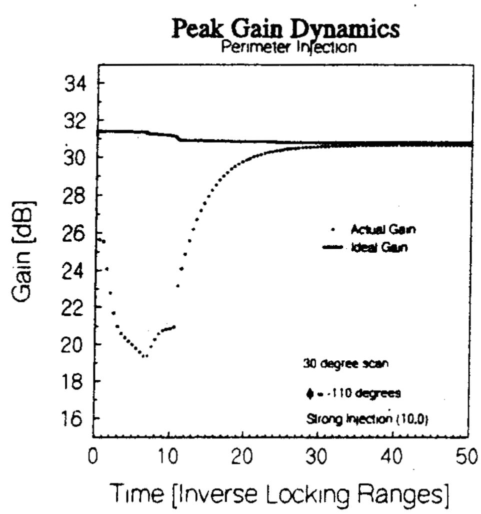

Figure 6. Antenna gain during beam steering for strong injection.

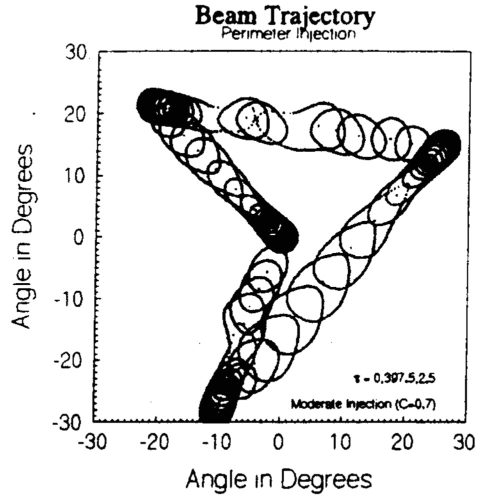

Figure 7. Sequential beam steering.